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New Approach for Stabilisation of Continuous Takagi Sugeno Fuzzy System

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(Abstract) This paper deals with the new approach for stabilization of continuous Takagi-Sugeno fuzzy models. Using a new Lyapunov function, new sufficient criteria are established in terms of Linear Matrix Inequality. First, a basic stability criterion is derived for open loop system. Next, a stabilization criterion with PDC controller and relaxed stability criterion are established. Finally, a new approach of PDC controller design is given.

Keywords: Takagi-Sugeno Fuzzy Ssystem; Linear Matrix Inequalities; Fuzzy Lyapunov Function; Parallel Distributed Controller PDC

1. INTRODUCTION

Fuzzy control systems have experienced a big growth of industrial applications in the recent decades, because of their reliability and effectiveness. Many researches are investigated on the Takagi-Sugeno models [4]–[5] which can combine the flexible fuzzy logic theory and rigorous mathematical theory into a unified framework. Thus, it becomes a powerful tool in approximating a complex nonlinear system.

Two classes of Lyapunov functions are used to analysis these systems: quadratic Lyapunov functions and non-quadratic Lyapunov ones which are less conservative than first class. Many researches are investigated with non-quadratic Lyapunov functions [3]–[11].

In this paper, a new stability conditions for Takagi Sugeno sufficient fuzzy models based on the use of fuzzy Lyapunov function are presented. This criterion is used with PDC controller to derive stabilization conditions and relaxed stability ones. These criteria are expressed in terms of Linear Matrix Inequalities (LMIs) which can be efficiently solved by using various convex optimization algorithms [12].

The organization of the paper is as follows. In section 2, we present the system description and problem formulation and we give some preliminaries which are needed to derive results. Section 3 will be concerned to stability analysis for continuous T-S fuzzy systems. Stabilization with PDC controller and relaxed stability criterion are given in section 4 and 5 respectively. Section 6 concerns the proposed approach to design a T-S fuzzy system with Parallel Distributed Controller (PDC). Finally section 7 makes conclusion.

Notation: Throughout this paper, a real symmetric matrix $S > 0$ denotes S being a positive definite matrix. The superscript ‘‘T’’ is used for the transpose of a matrix.

2. SYSTEM DESCRIPTION AND PRELIMINARIES

Consider a T-S fuzzy continuous model for a nonlinear system as follows:

$$\begin{array}{l} IF\ z_1(t)\ is\ M_{i1}\ and \ldots and\ z_p(t)\ is\ M_{ip} \\ THEN\quad \dot{x}(t) = A_i x(t) + B_i u(t)\quad i = 1,\ldots,r \end{array} \tag{1}$$

where $M_{ij}\left(i = 1,2,\ldots,r,\ j = 1,2,\ldots,p\right)$ is the fuzzy set and r is the number of model rules; $x(t) \in \Re^n$ is the state vector, $u(t) \in \Re^m$ is the input vector, $A_i \in \Re^{n\times n}$, $B_i \in \Re^{n\times m}$, and $z_1(t),\ldots,z_p(t)$ are known premise variables.

The final outputs of the fuzzy systems are:

$$\dot{x}(t) = \sum_{i=1}^{r} h_i\left(z(t)\right)\left\{A_i x(t) + B_i u(t)\right\} \tag{2}$$

where

$$z(t) = \left[z_1(t) z_2(t) \ldots z_p(t)\right]$$

$$h_i\left(z(t)\right) = w_i\left(z(t)\right) \Big/ \sum_{i=1}^{r} w_i\left(z(t)\right),$$

$$w_i\left(z(t)\right) = \prod_{j=1}^{p} M_{ij}\left(z_j(t)\right) \quad \text{for all } \underline{t}.$$

The term $M_{i1}\left(z_j(t)\right)$ is the grade of membership of $z_j(t)$ in M_{i1}

Since $\begin{cases} \sum_{i=1}^{r} w_i\left(z(t)\right) \succ 0 \\ w_i\left(z(t)\right) \geq 0, \qquad i = 1,2,\ldots,r \end{cases}$

We have $\begin{cases} \sum_{i=1}^{r} h_i\left(z(t)\right)=1 \\ h_i\left(z(t)\right)\geq 0, \qquad i=1,2,\ldots,r \end{cases}$ for all t.

The PDC fuzzy controller is represented by

$$u(t)=-\frac{\sum_{i=1}^{r} w_i\left(z(t)\right)F_i x(t)}{\sum_{i=1}^{r} w_i\left(z(t)\right)}=-\sum_{i=1}^{r} h_i\left(z(t)\right)F_i x(t) \quad (3)$$

The fuzzy controller design is to determine the local feedback gains F_i in the consequent parts.

The open-loop system is given by the equation (4)

$$\dot{x}(t)=\sum_{i=1}^{r} h_i\left(z(t)\right)A_i x(t) \quad (4)$$

By substituting (3) into (2), the closed-loop fuzzy system can be represented as:

$$\dot{x}(t)=\sum_{i=1}^{r}\sum_{j=1}^{r} h_i\left(z(t)\right)h_j\left(z(t)\right)\left\{A_i-B_iF_j\right\}x(t) \quad (5)$$

Lemma 1 [2]

If the number of rules that fire for all t is less than or equal to s, where $1\prec s\leq r$, then

$$\sum_{i=1}^{r} h_i^2\left(z(t)\right)-\frac{1}{s-1}\sum_{i=1}^{r}\sum_{i\prec\pi j}^{r} 2h_i\left(z(t)\right)h_j\left(z(t)\right)\geq 0,$$

where $\sum_{i=1}^{r} h_i\left(z(t)\right)=1, \qquad h_i\left(z(t)\right)\geq 0$

3. BASIC STABILITY CONDITIONS

Consider the open-loop system (6).

$$\dot{x}(t)=\sum_{i=1}^{r} h_i\left(z(t)\right)A_i x(t) \quad (6)$$

The aim of the next section is to find conditions for the stability of the unforced T-S fuzzy system by using the Lyapunov theory.

Theorem 1

The Takagi Sugeno fuzzy system (6) is stable if there exist positive definite symmetric matrices $P_k, k=1,2,\ldots,r$, such that the following LMIs hold.

$$P_k\succ 0, \quad k\in\left\{1,\ldots,r\right\} \quad (7)$$

$$\left\{A_i^T P_j+P_jA_i\right\}\prec 0, \quad i,j=1,\ldots,r \quad (8)$$

where $i,j=1,2,\ldots,r$

Proof

Let consider the Lyapunov function in the following form:

$$V\left(x(t)\right)=\sum_{k=1}^{r} x^T(t)P_k x(t)=\sum_{k=1}^{r}\varepsilon_k V_k\left(x(t)\right), \quad (9)$$

where

$V_k\left(x(t)\right)=x^T(t)P_k x(t); \; P_k=P_k^T$,and $P_k\succ 0, \; k=1,2,\ldots,r$

$\sum_{k=1}^{r}\varepsilon_k=1$

This candidate Lyapunov function satisfies

i) $V\left(x(t)\right)$ is C^1,

ii) $V(0)=0$ and $V\left(x(t)\right)\geq 0$ for $x(t)\neq 0$,

iii) $\left\|x(t)\right\|\to\infty\Rightarrow V\left(x(t)\right)\to\infty$.

The time derivative of $V\left(x(t)\right)$ with respect to t along the trajectory of the system (6) is given by:

$$\dot{V}\left(x(t)\right)=\sum_{k=1}^{r}\varepsilon_k\dot{V}_k\left(x(t)\right) \quad (10)$$

The equation (10) can be rewritten as,

$$\dot{V}\left(x(t)\right)=\sum_{k=1}^{r}\sum_{i=1}^{r} h_i\left(z(t)\right)\varepsilon_k x^T(t)\times\left\{A_i^T P_k+P_kA_i\right\}x(t) \quad (11)$$

Then,

$$\dot{V}\left(x(t)\right)=x^T(t)\sum_{k=1}^{r}\sum_{i=1}^{r} h_i\left(z(t)\right)\varepsilon_k\times\left\{A_i^T P_k+P_kA_i\right\}x(t)$$

If (8) holds, then $\dot{V}\left(x(t)\right)\prec 0$ and (6) is stable. This completes the proof.

4. STABILIZATION WITH PDC CONTROLLER

Consider the closed-loop system (5) which can be rewritten as

$$\dot{x}(t)=\sum_{i=1}^{r} h_i\left(z(t)\right)h_i\left(z(t)\right)G_{ii}x(t) + 2\sum_{i=1}^{r}\sum_{i\prec j} h_i\left(z(t)\right)h_j\left(z(t)\right)\left\{\frac{G_{ij}+G_{ji}}{2}\right\}x(t), \quad (12)$$

where

$$G_{ij}=A_i-B_iF_j.$$

In this section we define a fuzzy Lyapunov function and then consider stability conditions. A sufficient stability condition, for ensuring stability is given follows.

Theorem 2

The Takagi-Sugeno system (12) is stable if there exist positive definite symmetric matrices $P_k, k=1,2,\ldots,r$, and matrices $F_1,\ldots,F_r$ such that the following LMIs hold.

$$P_k\succ 0, \quad k\in\left\{1,\ldots,r\right\} \quad (13)$$

$$\left\{G_{ii}^T P_k+P_kG_{ii}\right\}\prec 0, \quad i,k\in\left\{1,\ldots,r\right\} \quad (14)$$

$$\left\{\frac{G_{ij}+G_{ji}}{2}\right\}^T P_k+P_k\left\{\frac{G_{ij}+G_{ji}}{2}\right\}\leq 0, \quad (15)$$

for $i,j,k=1,2,\ldots,r$ such that $i\prec j$

where

$$G_{ij} = A_i - B_i F_j$$

Proof

Let consider the Lyapunov function in the following form:

$$V\left(x\left(t\right)\right) = \sum_{k=1}^{r} \varepsilon_k V_k\left(x\left(t\right)\right) \tag{16}$$

with

$$V_k\left(x\left(t\right)\right) = x^T\left(t\right) P_k x\left(t\right), \qquad k = 1,2,\ldots,r$$

where $P_k = P_k^T$,and $P_k \geq 0, \quad k = 1,2,\ldots,r$.

The time derivative of $V\left(x(t)\right)$ is given by:

$$\begin{aligned} \dot{V}\left(x\left(t\right)\right) &= \sum_{k=1}^{r} \varepsilon_k \dot{V}_k\left(x\left(t\right)\right) \\ &= \sum_{k=1}^{r} \varepsilon_k \left\{\dot{x}^T\left(t\right) P_k x\left(t\right) + x^T\left(t\right) P_k \dot{x}\left(t\right)\right\} \end{aligned} \tag{17}$$

By substituting (12) into (17), we obtain,

$$\dot{V}\left(x\left(t\right)\right) = \Upsilon_1\left(x,z\right) + \Upsilon_2\left(x,z\right) \tag{18}$$

where

$$\begin{aligned} \Upsilon_1\left(x,z\right) &= x^T\left(t\right) \sum_{k=1}^{r} \sum_{i=1}^{r} h_i^2\left(z\left(t\right)\right) \cdot \varepsilon_k \\ &\times \left\{G_{ii}^T P_k + P_k G_{ii}\right\} x\left(t\right) \end{aligned} \tag{19}$$

$$\begin{aligned} \Upsilon_2\left(x,z\right) &= x\left(t\right)^T \sum_{k=1}^{r} \sum_{i=1}^{r} \sum_{i \prec j} h_i\left(z\left(t\right)\right) h_j\left(z\left(t\right)\right) \varepsilon_k \\ &\times \left\{\left(\frac{G_{ij} + G_{ji}}{2}\right)^T P_k + P_k \left(\frac{G_{ij} + G_{ji}}{2}\right)\right\}\Bigg] x\left(t\right) \end{aligned} \tag{20}$$

If (14) and (15) holds, the time derivative of the fuzzy Lyapunov function is negative. Consequently, we have

$$\begin{aligned} \dot{V}\left(x\left(t\right)\right) = x^T\left(t\right) \Bigg(& \sum_{k=1}^{r} \sum_{i=1}^{r} h_i^2\left(z\left(t\right)\right) \cdot \varepsilon_k \times \left\{\left(G_{ii}^T P_k + P_k G_{ii}\right)\right\} \\ & + \sum_{k=1}^{r} \sum_{i=1}^{r} \sum_{i \prec j} h_i\left(z\left(t\right)\right) h_j\left(z\left(t\right)\right) \varepsilon_k \\ & \times \left(\left\{\frac{G_{ij} + G_{ji}}{2}\right\}^T P_k + P_k \left\{\frac{G_{ij} + G_{ji}}{2}\right\}\right)\Bigg) x\left(t\right) \\ & \prec 0 \end{aligned}$$

and the closed loop fuzzy system (5) is stable. This completes proof.

5. RELAXED STABILITY CONDITIONS

In this section we define a fuzzy Lyapunov function and then consider relaxed stability conditions. A sufficient stability condition, for ensuring relaxed stability of closed-loop system is given follows.

Theorem 3

Assume that the number of rules that fire for all t is less than or equal to *s*, where $1 \prec s \leq r$, then, the Takagi-Sugeno system (12) is stable if there exist positive definite symmetric matrices $P_k, k = 1,2,\ldots,r$ and positive semidefinite matrices $Q_1,\ldots,Q_r$ such that the following LMIs hols.

$$G_{ii}^T P_k + P_k G_{ii} + \left(s-1\right) Q_k \prec 0, \quad i,k \in \left\{1,\ldots,r\right\} \tag{21}$$

$$\left\{\frac{G_{ij} + G_{ji}}{2}\right\}^T P_k + P_k \left\{\frac{G_{ij} + G_{ji}}{2}\right\} - Q_k \leq 0, \tag{22}$$

for $k = 1,2,\ldots,r$ and $i \prec j$

where

$s \succ 1$

$G_{ij} = A_i - B_i F_j$

Proof

Let consider the Lyapunov function in the following form:

$$V\left(x\left(t\right)\right) = \sum_{k=1}^{r} \varepsilon_k V_k\left(x\left(t\right)\right) = \sum_{k=1}^{r} \varepsilon_k x^T\left(t\right) P_k x\left(t\right) \tag{23}$$

where $P_k = P_k^T$,and $P_k \succ 0, \quad k = 1,2,\ldots,r, \quad \sum_{k=1}^{r} \varepsilon_k = 1.$

The time derivative of $V\left(x(t)\right)$ is given by:

$$\begin{aligned} \dot{V}\left(x\left(t\right)\right) &= \sum_{k=1}^{r} \varepsilon_k \dot{V}_k\left(x\left(t\right)\right) \\ &= \sum_{k=1}^{r} \varepsilon_k \left(\dot{x}^T\left(t\right) P_k x\left(t\right) + x^T\left(t\right) P_k \dot{x}\left(t\right)\right) \end{aligned} \tag{24}$$

By substituting (12) into (24), we obtain,

$$\dot{V}\left(x\left(t\right)\right) = \Upsilon_1\left(x,z\right) + \Upsilon_2\left(x,z\right) \tag{25}$$

where

$$\Upsilon_1\left(x,z\right) = x^T\left(t\right) \sum_{k=1}^{r} \sum_{i=1}^{r} h_i^2\left(z\left(t\right)\right) \varepsilon_k \times \left\{G_{ii}^T P_k + P_k G_{ii}\right\} x\left(t\right)$$

(26)

$$\begin{aligned} \Upsilon_2\left(x,z\right) &= x\left(t\right)^T \sum_{k=1}^{r} \sum_{i=1}^{r} \sum_{i \prec j} h_i\left(z\left(t\right)\right) h_j\left(z\left(t\right)\right) \cdot \varepsilon_k \\ &\times \left\{\left(\frac{G_{ij} + G_{ji}}{2}\right)^T P_k + P_k \left(\frac{G_{ij} + G_{ji}}{2}\right)\right\}\Bigg] x\left(t\right) \end{aligned} \tag{27}$$

If (22) holds, and from Lemma 3, we have

$$\begin{aligned} \dot{V}\left(x\left(t\right)\right) &\leq x^T\left(t\right) \left(\sum_{k=1}^{r} \sum_{i=1}^{r} h_i^2\left(z\left(t\right)\right) \cdot \varepsilon_k \times \left\{\left(G_{ii}^T P_k + P_k G_{ii}\right)\right\} \right) x\left(t\right) \\ & \quad + \sum_{k=1}^{r} \sum_{i=1}^{r} \sum_{i \prec j} 2 h_i\left(z\left(t\right)\right) h_j\left(z\left(t\right)\right) \cdot \varepsilon_k \; x^T\left(t\right) Q_k x\left(t\right) \\ & \leq \sum_{k=1}^{r} \sum_{i=1}^{r} h_i^2\left(z\left(t\right)\right) \cdot \varepsilon_k \times x^T\left(t\right) \left\{G_{ii}^T P_k + P_k G_{ii}\right\} x\left(t\right) \\ & \quad + \left(s-1\right) \sum_{k=1}^{r} \sum_{i=1}^{r} h_i^2\left(z\left(t\right)\right) \cdot \varepsilon_k \; x^T\left(t\right) Q_k x\left(t\right) \\ & = \sum_{k=1}^{r} \sum_{i=1}^{r} h_i^2\left(z\left(t\right)\right) \cdot \varepsilon_k \times x^T\left(t\right) \left\{G_{ii}^T P_k + P_k G_{ii} + \left(s-1\right) Q_k\right\} x\left(t\right) \end{aligned}$$

If (21) holds then the time derivative of the fuzzy Lyapunov

function is negative and the closed loop fuzzy system (5) is stable. This terminates the proof.

6. DESIGN OF PDC CONTROLLER

This section describes the PDC controller design for system (2) with control law(3), by determine F_i gains.

6.1. Main Design Approach

Consider condition stability given by Theorem 2. We define $X_k = P_k^{-1}$ by multiplying the inequality on the left and right by $X_k = P_k^{-1}$, we obtain the following LMIs conditions that constitute a stable fuzzy controller design problem

$$X_k \succ 0, \quad k \in \{1,\dots,r\} \tag{28}$$

$$X_k A_i^T + A_i X_k - X_k F_i^T B_i^T - B_i F_i X_k \prec 0, \quad i,k \in \{1,\dots,r\} \tag{29}$$

$$X_k A_i^T + A_i X_k + X_k A_j^T + A_j X_k - X_k F_j^T B_i^T - B_i F_j X_k - X_k F_i^T B_j^T - B_j F_i X_k \le 0.$$

for $i, j, k = 1, 2, \dots, r$ such that $i \prec j$

(30)

Let define $M_{ik} = F_i X_k$, and suppose that we have the following assumption

$$M_{i\alpha} = M_{i\beta} \cdot X_\beta^{-1} \cdot X_\alpha \qquad \text{for } \alpha, \beta = 1,\dots,r \tag{31}$$

The F_i gains are deduced through equation $F_i = M_{ik} X_k^{-1}$ for $i = 1,\dots,r$. Substituting into the above inequalities, we obtain:

$$X_k A_i^T + A_i X_k - M_{ik}^T B_i^T - B_i M_{ik} \prec 0, \quad i,k \in \{1,\dots,r\} \tag{32}$$

$$X_k A_i^T + A_i X_k + X_k A_j^T + A_j X_k - M_{jk}^T B_i^T - B_i M_{jk} - M_{ik}^T B_j^T - B_j M_{ik} \le 0. \tag{33}$$

for $i, j, k = 1, 2, \dots, r$ such that $i \prec j$

The stable fuzzy controller design problem in term of LMI conditions is given as

Find $X_k \succ 0$ and M_{ik} $(i, k = 1,\dots,r)$ satisfying

$$-X_k A_i^T - A_i X_k + M_{ik}^T B_i^T + B_i M_{ik} \succ 0, \tag{34}$$

$$-X_k A_i^T - A_i X_k - X_k A_j^T - A_j X_k + M_{jk}^T B_i^T + B_i M_{jk} + M_{ik}^T B_j^T + B_j M_{ik} \ge 0. \tag{35}$$

$i \prec j$

where

$$\begin{cases} X_k = P_k^{-1}, \;\; M_{ik} = F_i X_k, \\ M_{i\alpha} = M_{i\beta} \cdot X_\beta^{-1} \cdot X_\alpha \qquad \text{for} \quad \alpha, \beta = 1,\dots,r \end{cases}$$

The feedback gains F_i and P_k matrix can be obtained as

$$P_k = X_k^{-1}, \;\; F_i = M_{ik} X_k^{-1}$$

From the solutions X_k and M_{ik}.

6.2. Fuzzy Controller Design Using Relaxed Stability Conditions

Consider relaxed condition stability given by Theorem 3. The design problem to determine gains F_i for continuous fuzzy system is given as

$X_k \succ 0, Y_k \ge 0$, and M_{ik} $(i, k = 1,\dots,r)$ satisfying

$$-X_k A_i^T - A_i X_k + M_{ik}^T B_i^T + B_i M_{ik} - (s-1) Y_k \succ 0, \tag{36}$$

$$2Y_k - X_k A_i^T - A_i X_k - X_k A_j^T - A_j X_k + M_{jk}^T B_i^T + B_i M_{jk} + M_{ik}^T B_j^T + B_j M_{ik} \ge 0. \tag{37}$$

for $i \prec j$

where

$$\begin{cases} X_k = P_k^{-1}, \; M_{ik} = F_i X_k, Y_k = X_k Q_k X_k \\ M_{i\alpha} = M_{i\beta} \cdot X_\beta^{-1} \cdot X_\alpha \qquad \text{for} \quad \alpha, \beta = 1,\dots,r \end{cases}$$

The feedback gains F_i, matrices P_k, and Q_k can be obtained as

$$P_k = X_k^{-1}, \;\; F_i = M_{ik} X_k^{-1}, \; Q_k = P_k Y_k P_k \tag{39}$$

6.3. Decay Rate

The condition that $\dot{V}(x(t)) \le \alpha V(x(t))$ for all trajectories is equivalent to

$$G_{ii}^T P_k + P_k G_{ii} + 2\alpha P_k \prec 0 \tag{40}$$

For all i, k and

$$\left(\frac{G_{ij} + G_{ji}}{2}\right)^T P_k + P_k \left(\frac{G_{ij} + G_{ji}}{2}\right) + 2\alpha P_k \le 0 \tag{41}$$

for $i \prec j, \alpha \succ 0$

The largest lower bound on the decay rate can be found by resolve the problem

$$\underset{X_1,\dots,X_r, M_1,\dots,M_r}{\text{maximize}} \;\; \alpha$$

Subject to

$$X_k \succ 0, \quad -X_k A_i^T - A_i X_k + M_{ik}^T B_i^T + B_i M_{ik} - 2\alpha X_k \succ 0, \tag{42}$$

$$-X_k A_i^T - A_i X_k - X_k A_j^T - A_j X_k + M_{jk}^T B_i^T + B_i M_{jk} + M_{ik}^T B_j^T + B_j M_{ik} - 4\alpha X_k \ge 0. \tag{43}$$

$i \prec j$

where

$$\begin{cases} X_k = P_k^{-1}, \; M_{ik} = F_i X_k \\ M_{i\alpha} = M_{i\beta} \cdot X_\beta^{-1} \cdot X_\alpha \qquad \text{for} \quad \alpha, \beta = 1,\dots,r \end{cases} \tag{44}$$

6.4. Decay Rate Controller Design Using Relaxed Stability Conditions

The condition that $\dot{V}(x(t)) \le \alpha V(x(t))$ for all trajectories is equivalent to

$$G_{ii}^T P_k + P_k G_{ii} + (s-1) Q_k + 2\alpha P_k \prec 0 \tag{45}$$

For all i, k and

$$\left(\frac{G_{ij}+G_{ji}}{2}\right)^T P_k + P_k\left(\frac{G_{ij}+G_{ji}}{2}\right) - Q_k + 2\alpha P_k \le 0 \quad (46)$$

The largest lower bound on the decay rate can be found by resolve the problem

$$\underset{X_1,\ldots,X_r,M_1,\ldots,M_r}{\text{maximize}}\ \alpha$$

Subject to

$$X_k \succ 0, Y_k \ge 0 \quad (47)$$

$$-X_k A_i^T - A_i X_k + M_{ik}^T B_i^T + B_i M_{ik} - (s-1)Y_k - 2\alpha X_k \succ 0, \quad (48)$$

where

$$\begin{cases} X_k = P_k^{-1},\ M_{ik} = F_i X_k,\ Y_k = X_k Q_k X_k \\ M_{i\alpha} = M_{i\beta}\cdot X_\beta^{-1}\cdot X_\alpha \quad \text{for} \quad \alpha,\beta = 1,\ldots,r \end{cases} \quad (49)$$

Remark: The condition $M_{i\alpha} = M_{i\beta}\cdot X_\beta^{-1}\cdot X_\alpha$, for $\alpha,\beta = 1,\ldots,r$ is necessary to determine the F_i gains, another approach take proportionality between state vectors in order to compute these gains [8].

7. CONCLUSION

This paper provided a new non-quadratic fuzzy Lyapunov function for the stability and stabilization of Takagi-Sugeno fuzzy systems in terms of a combination of the LMI approach and Lyapunov theory. A relaxed stability conditions are derived by the use of the proposed approach. In addition, a new approach to design a T-S fuzzy system with Parallel Distributed Controller (PDC) is presented which is based on relation of proportionality between $M_{\alpha\beta}$ matrices in order to determine closed loop gains.

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